Manitoba Rural Learning Consortium

Knowledge Domain

Rational Number

(Grades 1-8)

mRLC

Knowledge Domain: Rational Number

Mathematics Grades 1-8

Dianne Soltess *St.James School Division*

Meagan Mutchmor *Winnipeg School Division*

© 2015 mRLC – Manitoba Rural Learning Consortium

Introduction

This draft document is intended to support teachers in planning, assessing, and reporting students' mathematics progress. This domain-focused document has been developed using the most current and relevant research related to methodology and pedagogy, the Manitoba Curriculum, and other related resources. The document compliments the mRLC Essential Learning and Backward Planning Templates all of which are meant to assist teachers in providing quality mathematics education to all students.

Purpose

The Knowledge Domain documents (for Grades1 – 8) provide teachers with a professional tool that describes the development of the following areas of mathematics:

Counting & Place Value
Addition and Subtraction
Multiplication and Division
Rational Number

The Knowledge Domains can be used separately when focusing on a topic of study, or they naturally merge with each other to assist teachers in scaffolding student instruction when planning for learning.

The acquisition of the concepts in each knowledge domain are developed by highlighting developmental benchmarks, providing the pedagogical knowledge needed in order to teach them along with illustrated examples of the outcomes.

Not all of our students are meeting grade-level outcomes. When documenting learning for the MB Report Card, a Level 4 is attained when students demonstrate their ability and understanding with the strategies indicated while working within the numeral range for their grade level.

Effective teachers of mathematics need to know the content knowledge of their subject area (what to teach) as well as the pedagogical content knowledge of their area (how to teach). This document provides teachers with the grade level outcomes and developmental benchmarks for each knowledge domain. This document will assist teachers in making instructional decisions when planning as well as providing them with insight in order to help them reflect on student progress when assessing.

This document will assist teachers in planning for these struggling learners through the use of the developmental benchmarks and the illustrated strategies.

Using the Knowledge Domains – Professional Learning Communities

The Knowledge Domain document is intended to provide teachers with a conceptual tool that they can use to think constructively about mathematics. Teachers can work individually, in small groups, as a staff, or across a school division to:

explore key understandings in each knowledge domain

connect to current research

develop an awareness of models and tools for teaching

reflect on the thinking behind the mathematics

explore a domain in a deeper realm

discuss student progress related to the developmental benchmarks

compare work samples against the benchmarks

help identify misconceptions

develop common assessments to assess student progress related to the benchmarks

© 2015 mRLC – Manitoba Rural Learning Consortium

Things to consider when teaching...Rational Number

Let's clear up some misconceptions! Fractions are your friend!

When asked, most students in grade 6 would define a fraction as a number less than 1. This is a misunderstanding that leads to future difficulty for them as learners when given the more difficult concepts in middle years. This same misconception also exists for decimal numbers.

A great first experience for students with fractions is in the context of fair shares. Context: 2 chocolate bars between 3 students
They begin to think of the numerator as the number of things being shared (2 chocolate bars) and the denominator as the number of shares needed (3 people).
Each person would get 2/3 of a chocolate bar. This is a difficult concept and must be developed overtime in meaningful real world contexts.

Some students have difficulty with rational number because they use whole number thinking in a rational number context. When comparing fractions for example they may think 1/10 is larger than 1/4 because in whole numbers 10 comes after 4 in the counting sequence and therefore is thought of as a larger number. It is important when first introducing fractions to students that we watch for indicators of fragile understanding. This is also a common error in student's ordering of decimal numbers when treated as if they were whole numbers, students may claim 0.56 is greater than 0.7 because 56 is greater than 7. Many students with fragile place value knowledge will often think the decimal with the greatest number of digits is the largest value. E.G. 0.764 & 0.1764, think 0.1764 is larger .

Another barrier to understanding...If students have a fragile knowledge of multiplication and believe that when you multiply…things get "bigger" or "more" Then understanding multiplication of fractions produces a challenge.

When teaching rational number link fractions, decimals and percents whenever possible rather than teaching them as separate topics.

© 2015 mRLC – Manitoba Rural Learning Consortium

Development of Rational Number Grades 1-8

Fractions underpin the development of proportional reasoning and are important for future mathematics study including that of algebra and probability.

D.Clarke, A.Roche, A. Mitchell, NCTM,Vol.13,No7,2008

The development of a student's knowledge of fractions begins in elementary school and progresses into middle school
In order for students to be able to enter into the middle years curriculum they need to transition from solving problems additively to thinking multiplicatively.
Eventually the goal is for students to become proportional reasoners & understand the relationship between fractions, ratios and proportions.

Sowder et al, 1998

The intent of this document is to assist educators in designing a quality program involving rich discussions and experiences for our students when learning about rational number. Greater understanding of student conceptions and misconceptions about fractions, effective practice for teaching are critically important to improve classroom instruction for our learners.

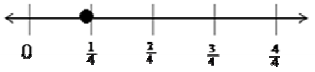
Whole Numbers

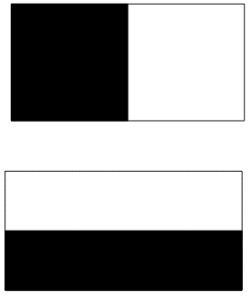
Is the set of numbers including the counting numbers 1,2,3,… & zero. {0,1,2,3….}
Whole numbers are not negative and are not fractions decimals or percents.

Rational Numbers

Are numbers that can be written as a fraction. in the form **a/b** where **a** and **b** are integers and **b≠0**. All rational numbers can be expressed as a terminating or repeating decimal.

FRACTIONS: Some key understandings that students should be exposed to as they develop their understanding of rational number.

The Whole Matters	Equivalence	Halving	Representations
In order to compare and discuss fractions in a meaningful way the whole must be the same. Students need to understand that in any fraction context the whole matters. Setting up real world problems helps students keep in mind the "whole". In a problem with "fair shares"...students can "half" cookies, but not pencils	Fractions can be re-named many different ways. You can rename **1/5** as **2/10** or **3/15** or **4/20** There are many names for a single point on the number line. $\frac{1}{2}$, **50%, 0.5, 50/100**	Students begin to use halving strategies when working with fractions. They learn that tenths can be made by halving fifths, sixths can be made by halving thirds, and eighths by halving fourths.	There are three representation for fractions; area, set, and measurement **AREA**  $\frac{3}{4}$ of the region is grey **SET**  $\frac{5}{6}$ of the set is black **MEASURE**  $\frac{1}{4}$ is indicated on the line

Congruence	The Bigger the Denominator the Smaller the Piece	Benchmark Fractions	Comparing Fractions	Ratio Tables
The pieces do not need to be the same shape (congruent) in order to be equivalent. Below, a both rectangles represent 1/2, the shaded regions are not the same shape but they are equivalent. 	Students need many hands-on experiences with many different materials and contexts to construct the understanding that the bigger the denominator the smaller the piece. This is a foundational idea about fractions that students need to understand and visualize.	Students need multiple experiences comparing fractions to **Benchmark Fractions.** Questions such as: Is $\frac{1}{4}$ closer to $\frac{1}{2}$ or 1? Which of these fractions is closer to 1, ($\frac{3}{4}$ or $\frac{2}{3}$) ? How do you know? Sometimes you need to compare the part that is missing $\frac{1}{4}$ & $\frac{1}{3}$ Which of these fractions is closer to 0, ($\frac{1}{6}$ or $\frac{1}{8}$) ? How do you know? Consider the size of each piece.	If the denominator is the same, when comparing fractions only the numerator needs to be considered. For example, when comparing $\frac{1}{4}$ and $\frac{3}{4}$ we only need to consider the numerator to determine which fraction is larger. *Think: "I have 1 of 4 pieces, or 3 of 4 pieces."* If the numerator is the same, when comparing fractions only the denominator needs to be considered. For example, when comparing $\frac{1}{2}$ and $\frac{1}{4}$ we only need to consider the denominator to determine which fraction is larger. *Think: "I have 1 of 2 pieces, or 1 of 4 pieces."*	The use of ratio tables is an effective strategy for students to use when problem solving with real world contexts. Students create a series of equivalent number pairs, to solve problems/ mental math warm ups. E.g. 65% of the theater tickets were sold. 100% 50% 25% 75% 30% 10% 540 270 135 405 Or a student can determine a percentage of the whole. 75% of the tickets were sold. Part 600 300 150 75 Whole 800 400 200 100

© 2015 mRLC – Manitoba Rural Learning Consortium

Rational Number Curricular Outcomes By Grade				
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**
In the early grades exposure to understanding of fair share and "half" are pre-requisite learnings for working with Fractions. Many students in primary grades understand that if at snack time there are 3 half apple pieces left in the bowl there is an apple and a half leftover.			**3.N.12**. Demonstrate an understanding of division by: representing and explaining division using equal sharing and equal grouping **3.N.13.**Demonstrate an understanding of fractions by: explaining that a fraction represents a portion of a whole divided into equal parts describing situations in which fractions are used comparing fractions of the same whole with like denominators	**4.N.8.** Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: *name and record fractions for the parts of a whole or a set *compare and order fractions *model and explain that for different wholes, two identical fractions may not represent the same quantity *provide examples of where fractions are used **4.N.9.** Describe and represent decimals (tenths and hundredths), C,P,S **4.N.10**.Relate decimals to fractions (to hundredths) **4.N.11.** Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: *using compatible numbers *estimating sums and differences *using mental math strategies to solve problems.
Other connections within Mathematics (Supporting Conceptual Understanding)				
			3.N.2. Represent and describe numbers to 1000, concretely, pictorially, and symbolically. [C, CN, V] • (AI) Read a 3-digit numeral without using the word "and" (e.g., 321 is three hundred twenty-one, NOT three hundred AND twenty-one) **3.SS.3** Demonstrate an understanding of measuring length (cm, m)	**4.N.5** Describe and apply mental math strategies such as : • Skip counting • Using doubling and halving • Using doubling and adding one more group • Use repeated doubling

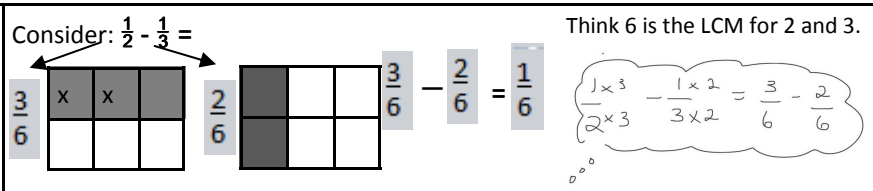
© 2015 mRLC – Manitoba Rural Learning Consortium

Rational Number Curricular Outcomes By Grade			
Grade 5	**Grade 6**	**Grade 7**	**Grade 8**
5.N.6. (AI) Explain that the interpretation of a remainder depends on the context: *ignore the remainder (e.g., making teams of 4 from 22 people) *round up the quotient (e.g., the number of five passenger cars required to transport 13 people) *express remainders as fractions (e.g., five apples shared by two people) *express remainders as decimals (e.g., measurement or money) **5.N.7.** Demonstrate an understanding of fractions by using concrete and pictorial representations to: *create sets of equivalent fractions *compare fractions with like and unlike denominators **5.N.8** Describe and represent decimals 1/10th equivalent to 1/100th 1/1000th **5.N.9**. Relate decimals to fractions (tenths, hundredths, thousandths). **5.N.10**.Compare and order decimals (tenths, hundredths, thousandths) by using: *benchmarks *place value *equivalent decimals **5.N.11.** Demonstrate an understanding of addition and subtraction of decimals (to thousandths), concretely, pictorially, and symbolically, by: *using personal strategies *using the standard algorithms *using estimation *solving problems	**6.N.1. D**emonstrate an understanding of place value for numbers: *greater than one million *less than one-thousandth **6.N.4**. Relate improper fractions to mixed numbers. **6.N.5**. Demonstrate an understanding of ratio, concretely, pictorially, and symbolically. **6.N.6.** Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically. **6.N.8.** Demonstrate an understanding of multiplication and division of decimals (involving 1-digit whole-number multipliers, 1-digit natural number divisors, and multipliers and divisors that are multiples of 10), concretely, pictorially, and symbolically, by: *using personal strategies *using the standard algorithms *using estimation *solving problems	**7.N.2.** Demonstrate an understanding of the addition, subtraction**, multiplication**, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, technology could be used). **7.N.3.** Solve problems involving percents from 1% to 100%. **7.N.4.** Demonstrate an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions **7.N.5.** Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences). **7.N.7**. Compare and order fractions, decimals (to thousandths), and integers by using: *benchmarks *place value *equivalent fractions and/or decimals **7.SP.4.** Express probabilities as ratios, fractions, and percents	**8.N.3.** Demonstrate an understanding of per cents greater than or equal to 0%. **8.N.4.** Demonstrate an understanding of ratio and rate. **8.N.5**. Solve problems that involve rates, ratios, and proportional reasoning. **8.N.6.** Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, (C,P,S) **8.N.8.** Solve problems involving positive rational numbers
Other connections within Mathematics (Supporting Conceptual Understanding)			
5.N.4 Mental math strategies: • Annex 0 • Halving and doubling • Distributive property **5.PR.2** single variable 1-step equations **5.SS.2** relationship between mm, cm and m **5.SS.3** Volume **5.SS.4** Capacity mL—L	**6.N.4.** Relate improper fractions to mixed numbers. **6.N.6.** Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically. **6.N.9.** Explain and apply the order of operations, excluding exponents (limited to whole numbers).	**7.PR.7**. Model and solve problems that can be represented by linear equations of the form: $ax + b = c$; $ax = b$; $x/a = b$, $a \neq 0$ concretely, pictorially, and symbolically, where *a*, *b*, and *c*, are whole numbers. **7.SS.1** Demonstrate an understanding of circles by: *describing the relationships among radius, diameter, and circumference of circles *relating circumference to pi (p) *determining the sum of the central angles *constructing circles with a given radius or diameter *solving problems involving the radii, diameters, and circumferences of circles	**8.PR.2.** Model and solve problems using linear equations of the form: $ax = b$ $x/a = b$, $a \neq 0$ $ax + b = c$ $x/a + b = c$, $a \neq 0$ $a(x + b) = c$ concretely, pictorially, and symbolically, where *a*, *b*, and *c*, are integers.

© 2015 mRLC – Manitoba Rural Learning Consortium

Conceptualising the Algorithm

Addition and Subtraction of Fractions: This can be an area that same students have difficulty with, especially if they do not have a good grasp of their basic facts, factors and multiples. Having a good grasp of these skills helps students determine equivalent denominators with ease. Students who struggle with determining

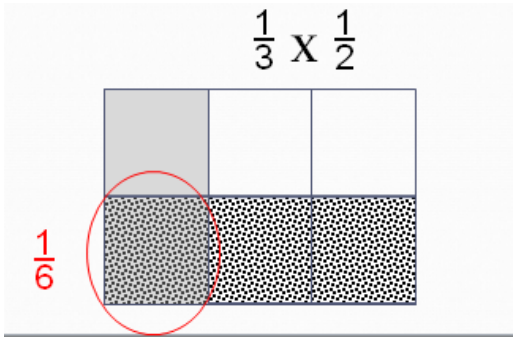


Multiplication of Fractions: This is can be modelled using the number line or the area model. Students would have used these models prior to grade 4 with whole number concepts.
Many students can follow the "rule" to multiply across the top and bottom….but it is important that they conceptualise that when multiplying fractions you actually get a smaller amount. This is a "big idea" and doesn't work if students use whole number thinking.

Consider the question $\frac{1}{3}$ x $\frac{1}{2}$ students should read this question as "One half of a group of one third"

Teaching multiplication of fractions using the area model helps learners understand and visualize the multiplicative relationship that is occurring.

- Draw rectangle and show thirds. Shade $\frac{1}{3}$
- Divide shape into halves. Shade $\frac{1}{2}$
(you should have 6 regions)
- Identify the region that is common to both shadings $\frac{1}{6}$



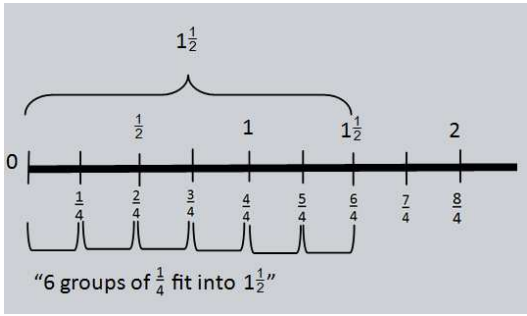
Division of Fractions: Can be linked to area model or number line.

Consider the question $1\frac{1}{2} \div \frac{1}{4}$

Many students know the "rule" to convert the improper fraction to $\frac{3}{2}$ and then use the equation $\frac{3}{2} \div \frac{1}{4}$ and "invert and multiply" $\frac{3}{2}$ x $\frac{4}{1} = \frac{12}{2}$ or 6. BUT...how do we help students *understand* division of fractions?

Students need to think of the question as "How many groups of $\frac{1}{4}$ are there in $1\frac{1}{2}$?"

- Use the number line to mark off $1\frac{1}{2}$. Iterate all the "halves" on the top of the numberline.
- On the bottom of the number line mark of all the "fourths"
- Identify the number of "fourths that fit into $1\frac{1}{2}$. The answer is 6.. Have students connect the numbers in the visual model with the numbers in the algorithm.



© 2015 mRLC – Manitoba Rural Learning Consortium

Proportional Reasoning Further Learning

Proportional reasoning is a cornerstone of middle school mathematics, and manifests itself in high school math when students are reasoning about slope, scale and trigonometry. It is more than understanding calculations for rate and ratio. But conceptualising the relationship between values. If given the rate a car drives in one hour you can determine the distance it will travel in ten hours. Many students struggle with lessons in middle school math and science due to their fragile conceptual understanding of proportional reasoning.

Proportional reasoning is the ability to make comparisons using multiplicative thinking. Many middle years students still reason about problems using additive thinking, which interferes with their ability to think proportionally. The development of proportional reasoning takes time and is developed through a sound understanding of the concepts of multiplication and division.

Firstly, it is important to understand the meaning of ratio and proportion. Ratio is a descriptor of a situation using a comparison. Example the ratio of chairs to desks is 3 to 4 then we are comparing the number of chairs to the number of desks. If we know there are 35 pieces of furniture in the class then we know there are 15 chairs and 20 desks. Thinking about this situation students have to understand the ratio and the multiplicative relationship between the ratio and the proportional situation given.

Chairs	3	6	9	12	15
Desks	4	8	12	16	20

Proportional reasoning is also developed through rich explorations and understanding of fractions, decimals and percents. Also understanding the key concepts in scale drawings lends itself to learning about proportional reasoning. This is a difficult concept to teach and is challenging for students to fully master. It is important when planning for instruction to ensure that rich tasks are presented to students throughout the middle school years that allow them to develop a strong understanding of proportional reasoning.

Proportional reasoning is complex, and is not developed in a linear fashion. It permeates all aspects of mathematics and is worth further study about how to make meaningful connections in our lessons for our students.

© 2015 mRLC – Manitoba Rural Learning Consortium

MB Documents:

- Kindergarten to Grade 8 Mathematics: Manitoba Framework of Outcomes
- Grade level support documents

Other Resources:

Big Ideas by Marian Small
Teaching Student Centered Mathematics by John Van De Walle
First Steps in Mathematics: Number Sense: Whole and Decimal Numbers, and Fractions
Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers By Susan J Lamon

Web Resources:

Proportional Reasoning: Paying Attention to Proportional Reasoning: http:// www.publications.serviceontario.ca/ecom

Mathematics Reasoning Inventory: is an online formative assessment tool, the questions focus on whole numbers, Decimals and Fractions
https://www.mathreasoninginventory.com/Home/ReasoningFractions

Developing Effective Fractions Instruction for Kindergarten Through Grade 8 :
http://ies.ed.gov/ncee/wwc/pdf/practice_guides/fractions_pg_093010.pdf

Further Learning:

Fractions, Decimals and Percents are more complicated that they may seem at first glance. Consider the following examples:

- 75% of the room is covered with carpet
- Alan ate $\frac{3}{4}$ of a sandwich
- I live 0.75 km from school
- 3 parts lemon juice to 4 parts water
- 3 out of 4 dentist encourage their patients to visit twice a year.
- $\frac{3}{4}$

The above examples all pertain to the same relationship.