mRLC

Knowledge Domain: Multiplication

Mathematics Grades 1-8

Manitoba Rural Learning Consortium

Knowledge Domain

Multiplication

(Grades 1-8)

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Introduction

This draft document is intended to support teachers in planning, assessing, and reporting students' mathematics progress. This domain-focused document has been developed using the most current and relevant research related to methodology and pedagogy, the Manitoba Curriculum, and other related resources. The document compliments the mRLC Essential Learning and Backward Planning Templates all of which are meant to assist teachers in providing quality mathematics education to all students.

Purpose

The Knowledge Domain documents (for Grades1 – 8) provide teachers with a professional tool that describes the development of the following areas of mathematics:

Counting & Place Value
Place Value
Addition and Subtraction
Multiplication and Division
Rational Number

The Knowledge Domains can be used separately when focusing on a topic of study, or they naturally merge with each other to assist teachers in scaffolding student instruction when planning for learning.

The acquisition of the concepts in each knowledge domain are developed by highlighting developmental benchmarks, providing the pedagogical knowledge needed in order to teach them along with illustrated examples of the outcomes.

Not all of our students are meeting grade-level outcomes. When documenting learning for the MB Report Card, a Level 4 is attained when students demonstrate their ability and understanding with the strategies indicated while working within the numeral range for their grade level.

Effective teachers of mathematics need to know the content knowledge of their subject area (what to teach) as well as the pedagogical content knowledge of their area (how to teach). This document provides teachers with the grade level outcomes and developmental benchmarks for each knowledge domain. This document will assist teachers in making instructional decisions when planning as well as providing them with insight in order to help them reflect on student progress when assessing.

This document will assist teachers in planning for these struggling learners through the use of the developmental benchmarks and the illustrated strategies.

Using the Knowledge Domains – Professional Learning Communities

The Knowledge Domain document is intended to provide teachers with a conceptual tool that they can use to think constructively about mathematics. Teachers can work individually, in small groups, as a staff, or across a school division to:

explore key understandings in each knowledge domain

connect to current research

develop an awareness of models and tools for teaching

reflect on the thinking behind the mathematics

explore a domain in a deeper realm

discuss student progress related to the developmental benchmarks

compare work samples against the benchmarks

help identify misconceptions

develop common assessments to assess student progress related to the benchmarks

Things to consider when teaching...MULTIPLICATION

What is multiplication? It is more than repeated addition!

When asked most students and teachers will define multiplication as repeated addition, and while this is true it is not the ONLY way we want our students to think about multiplication.

Multiplication is one of the 4 operations, but is also a way of thinking. In early grades most of student work in the area of early multiplication experiences revolves around the understanding of equal groups. These equal groups can be counted in a variety of ways from counting by ones, using skip counting, repeated addition etc.

At the later grades students are introduced to multiplication as a way of thinking. Rate problems, measurement, even work with fractions involves multiplicative thinking. (5 is 1/3 of the set, how many in the set?...you probably multiplied 5x3)

As teachers, it is important for us to be aware of all the important understandings when teaching multiplication. We need to present all the models to students so that their understanding is built on a strong foundation. This document seeks to unpack the domain of multiplication into simple **"must haves"** for instruction. (see page 4)

The purpose of this document is to help teachers identify the instructional benchmark understandings that are important for students to understand. The key instructional ideas are shared here as a trajectory of knowledge for teachers to follow as they plan their instruction.

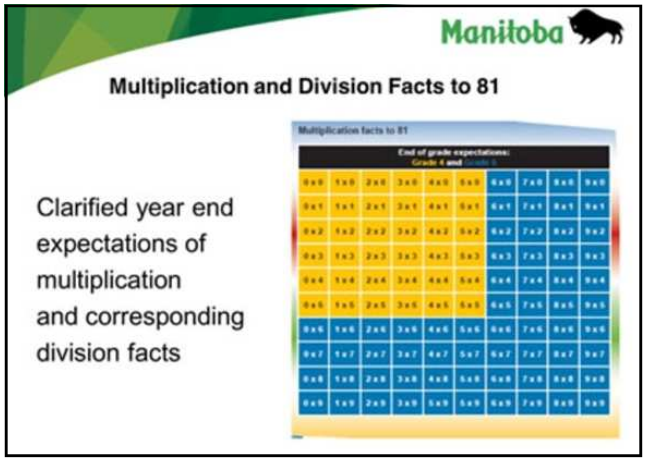
Students who think multiplicatively will have greater success constructing their understandings of the curriculum as they move through middle years and into high school.

Knowledge Domain: Multiplication

Grades 1-8

Provincial *End of Grade Level* expectations for

Multiplication and related Division facts

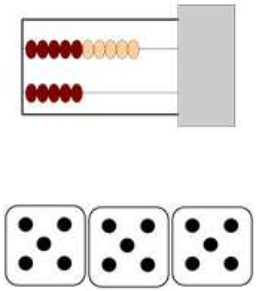
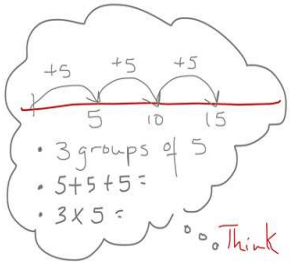
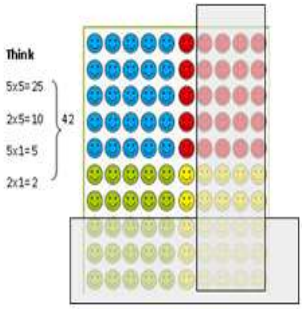
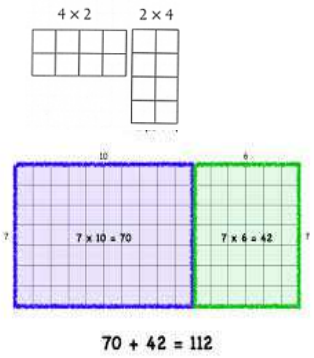
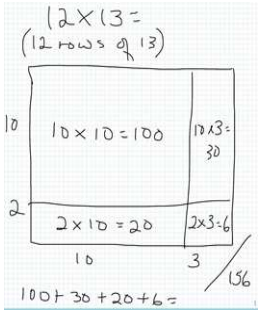
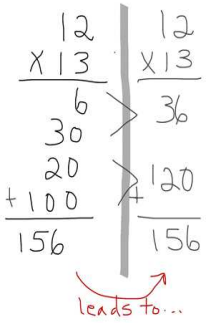


Once students have mastered the basic facts for addition and subtraction they can begin learning the basic facts for multiplication and division.

Using strategies that they know and linking them to the multiplication fact will help them realize that they know many of the facts already but just haven't connected them to the number sentence yet. E.g. Students who can skip count by fours, can easily use this to commit the facts for the 4s to memory.

Division fact knowledge can be acquired easily if connected to the inverse operation for multiplication.

Encourage students to explore patterns on the multiplication chart. Use of arrays on the chart can also be of assistance. For instance, 6x7=42, when traced on the multiplication table (tracking the 6 row and the 7 column) the 42 indicates the 42 squares in the area they traced.

	Equal Groups	Linear /Number line	Arrays	Area	Open Arrays	Algorithms
What is important.	Students first learn to form equal groupings. These equal groupings may be counted as individual items to determine total. Students progress to counting using repeated addition strategies to determine totals.	Once students understand equal groups, they can use their additive strategies to begin to think multiplicatively. Students should be encouraged to refrain from unit counting strategies, and progress to counting of groups.	An array is an organised arrangement of objects in rectangular formation. Once students work comfortably with equal groupings. They can begin to organise their groups into rows/arrays. Larger arrays can be created using two colours and benchmarks of 5 to assist with counting strategically.	Area model of multiplication is similar to the array model but using a rectangular grid. Use of the area model connects to shape and space strand with measurement. The array model lends itself nicely to teaching the commutative property. And leads to understanding the distributive property and partial products.	The multiplication table is based on the area model of multiplication. Open arrays are used to break up any array into workable units for multiplying. (partial products) Students then add partial products together.	Understanding open arrays leads gently into alternative algorithms, and eventually the standard algorithm.
Models						
Development/Understanding	Example: Three plates with 5 cookies on each plate. Counts: “1,2,3,4….15” Counts and adds: 5,and 5more is 10, 11,12,13...15” Counts “five, ten fifteen” Adds 5+5+5=	Student progresses to visualisation of the problem. Can think in groups (5) and keep track of the number of groups. (3)	Students can be introduced to the array. A blocking of the 100’s chart into groups of 5, and 25 assist students with determining their basic facts. Example: use the array to determine 6x7 by counting groups within the array.	Students progress from the array with items in the previous stage to the area model using measurement. In this stage students can draw the area of the multiplication scenario. As in the example above, this is useful in introducing the commutative property of multiplication.	Students continue to use the area model but no longer need to see the iteration of individual units and can visualize the “space” that each partial product creates.	Connection of the models can then be made to the algorithm. Students understand where the numbers in the standard algorithm come from and can relate to previously learned models.

Multiplication Curricular Outcomes By Grade				
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**
Establish strong counting skills Follow the guidance of the 5 counting principles	**1.N.1** Say the number sequence by 2s, 5s, 10s **1.N.3** Demonstrate an understanding of counting by using parts or equal groups to count sets **1.N.7** demonstrate concretely and pictorially how a number up to 30 can be represented by a variety of groups	**2.N.1** Say the number sequence from 0-100 by 2s, 5s, 10s	**3.N.1** Say the number sequence from 0-1000 **3.N11** demonstrate an understanding of multiplication to 5x5 **3.N.13** demonstrate and understanding of fractions	**4.N.4** Explain the properties of 0 &1 for multiplication and the property of 1 for division **4.N.5** Describe and apply mental math strategies such as : • Skip counting • Using doubling and halving • Using doubling and adding one more group • Use repeated doubling **4.N.6** demonstrate an understanding of multiplication
Other connections within Mathematics (Supporting Conceptual Understanding)				
Ability to subitize in a variety of models dice, finger patterns, five and ten frames		**2.PR.1** Predict an element in a repeating pattern using a variety of strategies **2.SS.4** Measure length to the nearest non-standard unit by : • Using multiple copies of a unit • Using a single copy of a unit (iteration process)	**3.SS.2** Relate the number of seconds to a minute, the number of minutes to an hour, the number of days to a month in a problem solving context **3.SS.3** Demonstrate an understanding of measuring length **3.SS.5** Demonstrate an understanding of perimeter	**4.N.3** (+) precursor for multiplicative thinking **4.PR.1** Identify and describe patterns found in tables and charts, including a multiplication table **4.SS.3** Demonstrate an understanding of area **4.SS.6** demonstrate an understanding of line symmetry (context of array and the mult. table) **4.SP.1** Demonstrate an understanding of many-to–one correspondence

Multiplication Curricular Outcomes By Grade			
Grade 5	**Grade 6**	**Grade 7**	**Grade 8**
5.N.2 Estimate a product **5.N.3** determine facts for 81 **5.N.4** Mental math strategies: • Annex 0 • Halving and doubling • Distributive property **5.N.5** multiplication (2 digit by 2 digit numbers)	**6.N.3**.Demonstrate an understanding of factors and multiples by determining multiples and factors of numbers less than 100 **6.N.5**. Demonstrate an understanding of ratio, concretely, pictorially, and symbolically. **6.N.6.** Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically.	**7.N.1**. Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0. **7.N.2.** Demonstrate an understanding of the addition, subtraction**, multiplication**, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, technology could be used).	**8.N.1**. Demonstrate an understanding of perfect squares and square roots, (C,P,S) **8.N.2.** Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). **8.N.4.** Demonstrate an understanding of ratio and rate. **8.N.5**. Solve problems that involve rates, ratios, and proportional reasoning. **8.N.6.** Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, (C,P,S) **8.N.7.** Demonstrate an understanding of multiplication
Other connections within Mathematics (Supporting Conceptual Understanding)			
5.N.1 Expanded notation **5.N.6** division (3 digit by 1 digit) inverse relationships **5.N.7** equivalent fractions (links to knowledge of multiples) **5.N.8** Describe and represent decimals 1/10th equivalent to 1/100th 1/1000th **5.N.10** order decimals 0.2, 0.02, 0.002 **5.PR.1** describe the relationship in a pattern or chart **5.PR.2** single variable 1-step equations **5.SS.2** relationship between mm, cm and m **5.SS.3** Volume **5.SS.4** Capacity mL—L	**6.N.4.** Relate improper fractions to mixed numbers. **6.N.6.** Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically. **6.N.9.** Explain and apply the order of operations, excluding exponents (limited to whole numbers). **6.PR.1.** Demonstrate an understanding of the relationships within tables of values to solve problems. **6.PR.3 (AI)** Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., $a + b = b + a$ or $a \times b = b \times a$). Describe the relationship in a table using a mathematical expression. Represent a pattern rule using a simple mathematical expression, such as $4d$ or $2n + 1$. **6.PR.4.** Demonstrate and explain the meaning of preservation of equality, concretely, pictorially, and symbolically. (mult) **6.SS.3.** Develop and apply a formula for determining the •perimeter of polygons •area of rectangles volume of right rectangular prisms	**7.N.3.** Solve problems involving per cents from 1% to 100%. **7.N.5** (AI) Simplify a positive fraction or mixed number by identifying the common factor between the numerator and denominator. **7.N.7.** Compare and order fractions, decimals (to thousandths), and integers by using benchmarks place value equivalent fractions and/or decimals **7.PR.1.** Demonstrate an understanding of oral and written patterns and their corresponding relations. **7.PR.2.** Construct a table of values from a relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. **7.PR.7**. Model and solve problems that can be represented by linear equations of the form: $ax + b = c$; $ax = b$; $x/a = b$, $a \neq 0$ concretely, pictorially, and symbolically, where a, b, and c, are whole numbers.	**8.N.3.** Demonstrate an understanding of per cents greater than or equal to 0%. **8.PR.2.** Model and solve problems using linear equations of the form: $ax = b$ $x/a = b$, $a \neq 0$ $ax + b = c$ $x/a + b = c$, $a \neq 0$ $a(x + b) = c$ concretely, pictorially, and symbolically, where a, b, and c, are integers. **8.SS.1.** Develop and apply the Pythagorean theorem to solve problems.

Glossary of Terms

Multiplication - A mathematical operation of combining groups of equal amounts; repeated addition; the inverse of division.

Product - The number obtained when two or more factors are multiplied

(e.g., in 6 × 3 = 18, 18 is the product).

5 x 6 = 30

Multiplicand Multiplier

Division - A mathematical operation involving two numbers that tells how many groups there are or how many are in each group.

Quotient - The answer to the division of two numbers

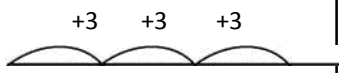
(e.g., in 12 ÷ 3 = 4, the quotient is 4).

30 ÷ 5 = 6

Dividend Divisor

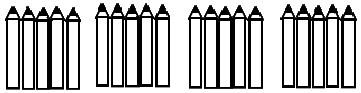
Meanings of Multiplication:

Repeated addition



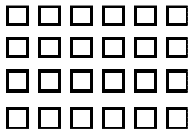
E.g. 3 + 3 + 3 = 9 Think: (skip counting) 3 , 6 , 9

Equal groups or sets
E.g. Pencils come in packages of 5.



How many pencils are in 4 packages?

An array
E.g. A classroom has 4 rows with 6 desks in each row.
How many desks are in the classroom?



Combination problems:
Marie wanted a single scoop ice-cream cone. She could get a waffle cone or a sugar cone. She could choose from chocolate, vanilla, rainbow or maple flavour.
How many possible ice-cream combinations could Marie choose from?

Rate problems:
John can bike 5kms / hour. If he bikes for two hours how far did he travel?
Vinh gets $10.50/hour at his job. He worked 22 hours this month. How much money did Vinh earn?

Multiplicative comparison problems
Rose has 15 pencils in her pencil case. Janet has three times as many pencils as Rose. How many pencils does she have?

Types of Division Problems/Questions

Partitive or equal sharing division

In this type of problem the total number of groups is known. The unknown is the number of items in each group. The quotient tells the amount for each share.

For example:
Jonas has 24 pieces of bubble gum.
He wants to share them equally among his 4 friends.
How many candies will each friend get?

Quotative, equal grouping or measurement division:

In this type of problem the number of items in each group is known. The unknown is the number of groups that can be made from the given quantity. The quotient tells how many groups can be made from the given quantity.

For example:
Martha bought cookies for 10¢ each.
She spent 90¢.
How many cookies did she buy?

Models	Manipulatives
Groups of	Counters
Rows of	Dotty arrays
Sets of	Base ten blocks
Arrays	Grid paper
Area	Dot cards
Open arrays	Screened dot cards
	rekenreks

Common multiplication and division problem solving types

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 5 = ?	**3 x ? = 15, and 15 ÷ 3 = ?**	**? x 5 = 15, and 15 ÷ 5 = ?**
Equal Groups	There are 3 baskets with 5 apples in each basket. How many apples are there in all? Measurement example: Pat has 3 pieces of licorice. Each piece is 5 cm long. How many cm of licorice does he have?	John wants share his 15 apples equally between 3 baskets. How many apples will be in each basket? Measurement example: Pat has 15 cm of ribbon which will be cut into 3 equal pieces. How long will each piece of ribbon be?	John has 15 apples. He wants to put 5 apples in each bag. How many bags will he need? Measurement example: Pat has 15 cm of ribbon, which she cuts into 5 cm pieces. How many pieces of ribbon will she have?
Arrays/Area	There are 3 rows of counters with 5 counters in each row. How many counters are there? Area example: Pat's garden is 3 m wide and 5 m long. What is the area of the garden?	If 15 counters are arranged into 3 equal rows, how many counters will be in each row? Area example: Pat's garden has an area of 15 square metres. If the garden is 3 m wide, how long is it?	If 15 counters are arranged into equal rows of 5 counters, how many rows will there be? Area example: Pat's garden has an area of 15 square metres. If the garden is 5 m long, how wide is it?
Compare	Pat's book costs $5. Mary's book costs 3 times as much as Pat's book. How much does Mary's book cost? Measurement example: An elastic band is 5 cm long. When it is stretched out it is 3 times as long. How long is the elastic band when it is stretched out?	Mary's book costs $15. That is 3 times the cost of Pat's book. How much does Pat's book cost? Measurement example: An elastic band is 15 cm long when it is stretched out. That is 3 times as long as it is when it is not stretched out. How long is the elastic band when it is not stretched out?	Mary's book costs $15 and Pat's book costs $5. How many times as much does Mary's book cost compared to Pat's book? Measurement example: An elastic band was 5 cm long before it was stretched out. Now it is stretched to be 15 cm long. How many times as long is the elastic band now then it was before it was stretched out?
General	a x b = ?	a x ? = c, and c ÷ a = ?	? x b = c, and c ÷ b = ?

Resources

MB Documents:

- Kindergarten to Grade 8 Mathematics: Manitoba Framework of Outcomes
- Grade level support documents

Other Resources:

Big Ideas by Marian Small

Teaching Student Centered Mathematics by John Van De Walle

Web resources:

New Zealand Website , www.nzmaths .co.nz (numeracy project materials, books, Teaching Multiplication and Division)

Further Learning:

Baroody 's Three Phase Process of Learning (2006)

Phase 1: Counting Strategies

Using objects (manipulatives) or verbal counting to determine answer

Phase 2: Reasoning Strategies

Using known information to logically determine an unknown combination

Phase 3: Producing answers efficiently (fast and accurately) "just knowing"

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