Manitoba Rural Learning Consortium

Knowledge Domain

Counting & Place Value

(Grades 1-8)

mRLC

Knowledge Domain: Counting & Place Value

Mathematics Grades 1-8

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Introduction

This draft document is intended to support teachers in planning, assessing, and reporting students' mathematics progress. This domain-focused document has been developed using the most current and relevant research related to methodology and pedagogy, the Manitoba Curriculum, and other related resources. The document compliments the mRLC Essential Learning and Backward Planning Templates all of which are meant to assist teachers in providing quality mathematics education to all students.

Purpose

The Knowledge Domain documents (for Grades1 – 8) provide teachers with a professional tool that describes the development of the following areas of mathematics:

Counting & Place Value
Addition and Subtraction
Multiplication and Division
Rational Number

The Knowledge Domains can be used separately when focusing on a topic of study, or they naturally merge with each other to assist teachers in scaffolding student instruction when planning for learning.

The acquisition of the concepts in each knowledge domain are developed by highlighting developmental benchmarks, providing the pedagogical knowledge needed in order to teach them along with illustrated examples of the outcomes.

Not all of our students are meeting grade-level outcomes. When documenting learning for the MB Report Card, a Level 4 is attained when students demonstrate their ability and understanding with the strategies indicated while working within the numeral range for their grade level.

Effective teachers of mathematics need to know the content knowledge of their subject area (what to teach) as well as the pedagogical content knowledge of their area (how to teach). This document provides teachers with the grade level outcomes and developmental benchmarks for each knowledge domain. This document will assist teachers in making instructional decisions when planning as well as providing them with insight in order to help them reflect on student progress when assessing.

This document will assist teachers in planning for these struggling learners through the use of the developmental benchmarks and the illustrated strategies.

Using the Knowledge Domains – Professional Learning Communities

The Knowledge Domain document is intended to provide teachers with a conceptual tool that they can use to think constructively about mathematics. Teachers can work individually, in small groups, as a staff, or across a school division to:

explore key understandings in each knowledge domain

connect to current research

develop an awareness of models and tools for teaching

reflect on the thinking behind the mathematics

explore a domain in a deeper realm

discuss student progress related to the developmental benchmarks

compare work samples against the benchmarks

help identify misconceptions

develop common assessments to assess student progress related to the benchmarks

Things to consider when teaching...Counting

Many students enter school with the ability to recite the number sequence, but they may have no connection to using counting as a mathematical tool. Students may or may not be able to connect the numbers they are reciting to symbolic representation of the numerals or what a given set or quantity may look like.

Students need many opportunities to count in their early years of schooling. Repeated opportunities to count should be structured to deliberately connect the quantity to the symbol and to the number word. Any time a student is asked to count forwards they should also be given the same number of opportunities to count backwards. One of the reasons students in later grades struggle with subtraction is that they are fragile in their knowledge of the backwards counting sequence.

The following routines/practices are important for giving students varied opportunities to count and solidify their understanding of all aspects of counting.

- Chants, counting songs and rhymes
- Connect actions to counting, clapping, jumping, etc.
- When singing or chanting, use objects to add or remove, "act out"
- Give them a variety of items to count, including standard and non-standard groupable models. Students should progress to counting, organising, modelling larger collections (between 100 –1000)
- Use the words same as, more than, fewer when counting to help students develop a sense of relative size of number
- Skip counting games (foundation for addition & multiplication)
- Students at the higher grades are not confident counting in larger numeral ranges. They struggle reciting 4-7 digit numbers with fluidity.

Knowledge Domain Counting & Place Value

Grades 1-8

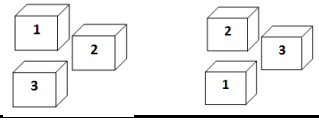
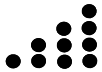
Counting is quite complex and is harder for students than we tend to think. It involves sequencing, ordering and hierarchical inclusion. (Any given quantity has other quantities "nested" within it.)
Meaningful counting is more than reciting the rote counting sequence. Comfort with the rote counting sequence (forward, backwards) & skip counting all contribute to a student learning to use and understand the counting of items and groups.
Counting involves understanding how numbers work, place value and the base ten system, the counting principles and how to use various counting strategies when problem solving.
The intent of this document is to outline key ideas and understandings that teachers should focus on to ensure that learners become confident counters.

As a student progresses in their understanding and use of counting strategies they:

- Acquire the counting principles
- Learn to subitize *(To determine the quantity of a small group of objects rapidly without counting)*
- Use Temporal /auditory counting
- Skip count, which links to multiplication, multiples,
- Learn to estimate
- Grapple with more and less and relative size of number
- Organize their counting and use strategies to refine counting skills
- Apply place value thinking
- Use positioning to count (ordinal numbers)
- Recognise numerical patterns
- Recognise the absence of items to count (empty set)
- Identify counting patterns in tables and charts

Counting Principles

In 1978, Rochel Gelman and Randy Gallistel introduced 5 principles that children need to understand in order to be considered competent counters. Many children learn to count without much assistance. However, the complexities of the 5 principles may not be developed in our learners unless they are formally introduced through direct instruction. As teachers of young children it is important to know how to deliberately expose students to the 5 counting principles as well as to be aware of what questions to ask children and what to look for when they are counting so that the understanding of counting is developed in depth. Developmentally most students should be proficient in their understanding of the counting principles by the end of grade 1.

The 1:1 Counting Principle	The Stable-Order Principle	The Cardinality Principle	The Abstraction Principle	The Order-Irrelevance Principle
When counting each object has only one number count. 1 2 3 4	The words we use when counting have an order that never changes. One, Two, Three, Four… Four, Three, Two, One	When counting a set of items, the last number "said" indicates how many items are in the set. "1 2 3 4 "	When counting it doesn't matter what we count, a quantity is always constant. 3 blocks or 3 blocks	When counting items, it doesn't mater what order you count them the total will always be the same. 
Teaching Strategies				
Students should have the rote counting sequence in place for the size of set being counted. Encourage students to physically touch or move objects when counting. Have them say one number name for each item moved. Temporal Counting: Dropping objects into a jar and having students say one number word for each object they "hear" being dropped. (eyes closed)	Knowledge of the rote counting sequence is vital to understand this principle. Point out to students the patterns in our number system (0-9 sequence) Use "subitizing" to formalize understanding of number. Students should understand that  is 5 before the symbol of 5 is attached. Students should order sets of objects, picture cards etc. to develop relative size of quantity understanding as well. 	Have students count out a set of items and place them in your hand. Ask: "How many counters are in my hand?" If a student needs to recount the items they do not have the cardinality principle in place. Have students count items placed in rows rather than groups, do they know that when they get to the end of the row, they have verbally said the number of items in the set? Match numbers to quantities, eventually move to groups of items vs. rows of items. (do students know when to stop counting?	This is a difficult concept for students to grasp. Hence the word "Abstract". Sometimes students think 5 elephants is *more than*, 5 bunnies. They think this because elephants take up more space. BUT...we want students to understand that 5 is always 5, no more no less. Whatever is being counted a set of five of anything will always have the name number of items.	Counting items in a variety of ways is important. Left to right Right to left Top to bottom Bottom to top This is one of the most misunderstood/fragile principles. Have students count out items onto a counting mat. Determine the number in the set. Then turn the placemat 1/2 turn. Ask: "How many items in the set now?"

Counting Curricular Outcomes By Grade				
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**
K.N.1. Say the number sequence by 1's, starting anywhere from 1 to 30 and from 10 to 1. **K.N.5**. Demonstrate an understanding of counting to 10 by: Indicating that the last number said identifies 'how many' Showing that any set has only one count	**1.N.1.** Say the number sequence by: 1s forward and backward between any two given numbers (0 to 100) 2s to 30, forward starting at 0. 5s and 10s to 100, forward starting at 0. **1.N.3** Demonstrate an understanding of counting by: Using the counting strategy Using parts or equal groups to count sets **1.N.10.** Describe and use mental mathematics strategies including counting back using one more, one less making 10 starting from known doubles using addition to subtract to determine the basic addition and related subtraction facts to 18. Recall of one more and one less, complementary (compatible) numbers that add up to 5 and 10, doubles (up to 5 + 5), and related subtraction facts is expected by the end of Grade 1.	**2.N.1.** Say the number sequence from 0 to 100 by: 2s, 5s, and 10s, forwards and backwards, using starting points that are multiples of 2, 5, and 10 respectively. 10s using starting points from 1 to 9. 2s starting from 1. **2.N.3.** Describe order or relative position using ordinal numbers. **2.N.5.** Compare and order numbers up to 100. **2.N.7.** Illustrate, concretely and pictorially, the meaning of place value for numbers up to 100.	**3.N.1**.Say the number sequence forward and backward from 0 to 1000 by: 10s, or 100s, using any starting point 5s using starting points that are multiples of five 25s using starting points that are multiples of 25 From 1 – 100 by: 3s, using starting points that are multiples of 3 4s, using starting points that are multiples of 4 **3.N.3.** Compare and order numbers to 1000 **3.PR.1.** Demonstrate an understanding of increasing patterns by: Describing, Extending, Comparing Creating, Patterns using manipulatives, diagrams, and numbers (to 1000) **3.PR.2.** Demonstrate an understanding of decreasing patterns by: Describing, Extending, Comparing Creating, Patterns using manipulatives, diagrams, and numbers (to 1000)	**4.N.2.** Compare and order numbers to 10 000 **4.N.5.** Describe and apply mental mathematics strategies, such as Skip-counting from a known fact.

Counting Curricular Outcomes By Grade			
Grade 5	**Grade 6**	**Grade 7**	**Grade 8**
5.N.1. Represent and describe whole numbers to 1 000 000 **5.N.7.** Demonstrate an understanding of fractions by using concrete and pictorial representations to: Create sets of equivalent fractions Compare fractions with like and unlike denominators	**6.N.7.** demonstrate and understanding of integers (C,P,S)	**7.N.7.** Compare and order fractions, decimals (to thousandths), and integers by using: Benchmarks Place value Equivalent fractions and/or decimals	
Other connections within Mathematics (Supporting Conceptual Understanding)			
5.N.3. Apply mental math strategies to determine multiplication and related division (9x9) **5.N.8.** Describe and represent decimals **5.N.10.** Compare and order decimals		**7.PR.1.** Demonstrate an understanding of oral and written patterns and their corresponding relations	

Things to consider when teaching...Place Value

Levels of Understanding Place Value

In 1989 Sharon Ross proposed a five stage development of place value understanding.

Stage 1 – The child associates two-digit numerals with the quantity they represent.
25 means the whole amount of 25

Stage 2 – The child identifies the positional names but do not necessarily know what each digit represents.
Uses positional labels only
In 58, there are 8 ones and 5 tens

Stage 3 – The child can identify the face value of digits in a number
In 24 the 2 means 2 tens and the 4 means 4 ones
The value of each digit may not be known

Stage 4 – In this stage a true understanding of place value occurs.
The child knows that the tens digit represents quantities of ten units
They can coordinate the part-whole relationship with two-digit numbers.

Stage 5 – The child understands the structure of our numeration system.
The child knows that digits in a two-digit numeral represent a partitioning of the whole quantity into tens and ones.
They know that the number represented is the sum of the parts.

Models for Teaching Place Value

Place Value Chart	Money (although more difficult without the penny)
Fingers	Tent cards
Cubes	Arrow cards
Beans	Base Ten blocks
Sticks/straws for bundling	Digi Blocks
Ten frames	Place value disks
Beaded number line	

Knowledge Domain Counting & Place Value

Grades 1-8

Place Value concepts are acquired while students are learning about counting and quantities, and making groups. While place value is listed explicitly in some curricular outcomes, it also permeates many topics in math. A deep understanding of place value and the base ten system is needed in order to develop fluency when calculating.

Prerequisite knowledge needed before beginning work on place value:

- Conservation of number
- Part-whole understanding
- Count on and back
- Count to 100
- Read and write numbers to 100
- Recognize that each number in the counting sequence represents an amount that is one more than the previous counting number
- Count by tens
- Recognize the 1-9 sequence in the decades
- Recognize the pattern of 1-9 within each decade
- Understand ten as a unit – unitize
- See the benefit of organizing into tens

Levels of Abstraction

Level 1 – Materials/objects can be grouped into a ten. Students can still see the ten ones but they have been grouped or bundled into a ten but they can also be unbundled to show the 10 ones.
Level 2 - The 10 ones have been pre-grouped and are unable to be unpacked however one 10 still looks like 10 ones, for example, 10 dots on a number strip.
Level 3 - A different looking 10 marker is used, for example, when using money, 10 x $1 coins can be traded for a $10 bill .
Level 4 - 10 is represented by virtue of its position, rather than its appearance.

Place Value Curricular Outcomes By Grade

Kindergarten	Grade 1	Grade 2	Grade 3
Pre-requisite understanding. Work on consolidation of counting principles.	**1.N.4.** Represent and describe numbers to 20 concretely, pictorially, and symbolically. **1.N.7**.Demonstrate, concretely and pictorially, how a number, up to 30, can be represented by a variety of equal groups with and without singles.	**2.N.4.** Represent and describe numbers to 100, concretely, pictorially, and symbolically. **2.N.5.** Compare and order numbers up to 100. **2.N.6**. Estimate quantities to 100 using referents. **2.N.7**. Illustrate, concretely and pictorially, the meaning of place value for numbers to 100.	**3.N.2**. Represent and describe numbers to 1000, concretely, pictorially, and symbolically. **3.N.3.** Compare and order numbers to 1000. **3.N.4.** Estimate quantities less than 1000 using referents. **3.N.5**. Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.
Grade 4	**Grade 5**	**Grade 6**	**Grade 7-8**
4.N.1. Represent and describe whole numbers to 10 000, pictorially and symbolically. **4.N.2.** Compare and order numbers to 10 000. **4.N.9.** Describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically.	**5.N.1**. Represent and describe whole numbers to 1 000 000. **5.N.8.** Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically. **5.N.10.** Compare and order decimals (tenths, hundredths, thousandths) by: *using benchmarks *place value *equivalent decimals.	6.N.1. Demonstrate and understanding of place value for numbers: *greater than one million *less than one thousandth **Note:** "Unitizing" Part of developing fluency in calculation processes is the ability to unitize this involves: • Exchanging ten items for one group of ten • Thinking of that one group of ten as though it is one object rather than one group comprised of ten other objects. The concept of unitizing should proceed any formal place value instruction. This should be in place by mid grade 2.	Students continue to consolidate understanding of the Base 10 system, in rational number contexts.

Counting & Place Value

Learning Progressions (from Least sophisticated to more sophisticated.) *Note that some students may skip certain steps as the progression is not defined by linear development.*

Kindergarten

- Pre-rote counting sequence
- Rote counting sequence
- One to one correspondence
- Concepts of more less and same
- Recognises pattern in the number system
- Decades and 0-9 sequence
- Rational counting: uses Rote counting sequence when counting items
- Uses pattern in conjunction with counting to skip count
  - 2s,5s,10s
  - 3s,4s
- Strategic counting
  - Count All
  - Count On, Count Back
  - Count On/Off Decades (±1,±10,±100)
- Counting by groups
- Double Counting (by ones and groups)

End of Grade 3

Next Steps:

Connect to other domain documents especially (+ & -)
Count using other methods
Count using different models
- Fractions
- Money
- Sets/groups using 15s, 25s etc.
- Use of other bases for counting can enhance place value understanding e.g. Base three

Common Misconceptions *(in no particular order)*

- Students may have some but not all the counting principles in place
- Have the rote counting sequence in place, but may not associate it with the objects they are counting, even if they have 1:1 correspondence
- Can count objects when placed in a line, struggles when placed in a group or bunch or in random fashion
- Can solve problems using the rote counting sequence, but must always start from 1 when adding collections. They are not able to count on.
- Can recite the rote counting sequence, but make errors when asked to count from random starting points. (forwards or backwards)
- Do not understand the role of 0 as a place holder when counting over decades (9 to 10, 19 to 20, 99 to 100) Struggle with counting on 1 more with numbers that end in a 9 E.g. 349
- When counting on or counting back they include the initial starting number. This is a common misconception when subtracting. E.g. When removing 3 items form 12 they count back 12, 11, 10...rather than hold 12 as the starting point in their head and say 11, 10, 9. Be mindful of students who repeatedly have answers that are "1 off"
- Can count on but does not connect this counting to the action of addition.
- Can use counting on and counting back reliably, but does not connect the use of this counting strategy to subtraction situations. E.g. 12-3 counts back all the way to 3 rather than count back 3 to get 9, or 12-9 counts back 9 rather than count up from 9 to 12.
- Can count on and back but does not associate this with addition or subtraction E.g. Can recite 14, 24, 34, 44 … but when given 14 + 10 = they may revert to counting on by ones from 14

Further Learning:

Place Value Understanding "Look Fors"

As students begin to understand the base ten system beyond counting items and think mathematically to solve problems by manipulating objects, sets and groups, they acquire a deeper number sense.

Look For…

- **Counting On:** once a student can count on from any given number they are showing that they do not rely on the rote counting sequence, and are emerging as a part-whole thinker. They are able to hold groups n their head as well as individual items. **Prior Knowledge:** Counting by ones, counting by groups, counting by groups and ones. **Next Step:** Connect this to naming two digit numbers by tens and ones. (24 is 2 tens or twenty and 4 ones or 4) follow with three digit numbers. Enhance place value knowledge by use of benchmark numbers of 5 and 10.

- **Part-Whole Thinking:** Additive: When students are able to think in groups and ones they can then begin to deepen their number sense by partitioning numbers as well as re-naming numbers. E.g. They can think of 27 as a standard partitioning or as 10 an 17 ones, or as 10 + 10 + 5 +2 or as 5 fives and 2. They can work with non-standard items in columns when using a place value chart and use the context of the problem to help them determine how best to partition the numbers they are working with. **Next Step**: Connect to Mental Math strategies as per the curriculum. Comfortable partitioning of number is a necessary skill in order to flexibly apply the strategies introduced in the curriculum.

- **Part-Whole Thinking:** Multiplicative: When students can use partitioning strategies with ease as well as solve problems with additive thinking strategies they begin to use place value and counting knowledge to strategically solve problems using multiplicative thinking. Prior understanding of non-standard representation of number can be extended to tenths and hundredths. (3.15 has a 1 in the tenths place but 31 tenths in total.) **Next Steps:** Students who are comfortable partitioning numbers and who think multiplicatively are showing readiness for introduction to the distributive property. **see Multiplication and Division Knowledge Domain document*

Resources

MB Documents:

- Kindergarten to Grade 8 Mathematics: Manitoba Framework of Outcomes
- Grade level support documents

Other Resources:

Big Ideas by Marian Small
Teaching Student Centered Mathematics by John Van De Walle
First Steps in Mathematics: Number Sense:

Web Resources:

New South Wales Numeracy Continuum (k-10)
http://www.numeracycontinuum.com/

New Zealand Website, www.nzmaths.co.nz (numeracy project materials, books, book 4 or book 5