Manitoba Rural Learning Consortium

Knowledge Domain

Addition & Subtraction

(Grades 1-8)

mRLC

Knowledge Domain: Addition and Subtraction

Mathematics Grades 1-8

1. Acknowledgements
2. Introduction
3. Content

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Introduction

This draft document is intended to support teachers in planning, assessing, and reporting students' mathematics progress. This domain-focused document has been developed using the most current and relevant research related to methodology and pedagogy, the Manitoba Curriculum, and other related resources. The document compliments the mRLC Essential Learning and Backward Planning Templates all of which are meant to assist teachers in providing quality mathematics education to all students.

Purpose

The Knowledge Domain documents (for Grades1 – 8) provide teachers with a professional tool that describes the development of the following areas of mathematics:

Counting & Place Value
Addition and Subtraction
Multiplication and Division
Rational Number

The Knowledge Domains can be used separately when focusing on a topic of study, or they naturally merge with each other to assist teachers in scaffolding student instruction when planning for learning.

The acquisition of the concepts in each knowledge domain are developed by highlighting developmental benchmarks, providing the pedagogical knowledge needed in order to teach them along with illustrated examples of the outcomes.

Not all of our students are meeting grade-level outcomes. When documenting learning for the MB Report Card, a Level 4 is attained when students demonstrate their ability and understanding with the strategies indicated while working within the numeral range for their grade level.

Effective teachers of mathematics need to know the content knowledge of their subject area (what to teach) as well as the pedagogical content knowledge of their area (how to teach). This document provides teachers with the grade level outcomes and developmental benchmarks for each knowledge domain. This document will assist teachers in making instructional decisions when planning as well as providing them with insight in order to help them reflect on student progress when assessing.

This document will assist teachers in planning for these struggling learners through the use of the developmental benchmarks and the illustrated strategies.

Using the Knowledge Domains – Professional Learning Communities

The Knowledge Domain document is intended to provide teachers with a conceptual tool that they can use to think constructively about mathematics. Teachers can work individually, in small groups, as a staff, or across a school division to:

explore key understandings in each knowledge domain

connect to current research

develop an awareness of models and tools for teaching

reflect on the thinking behind the mathematics

explore a domain in a deeper realm

discuss student progress related to the developmental benchmarks

compare work samples against the benchmarks

help identify misconceptions

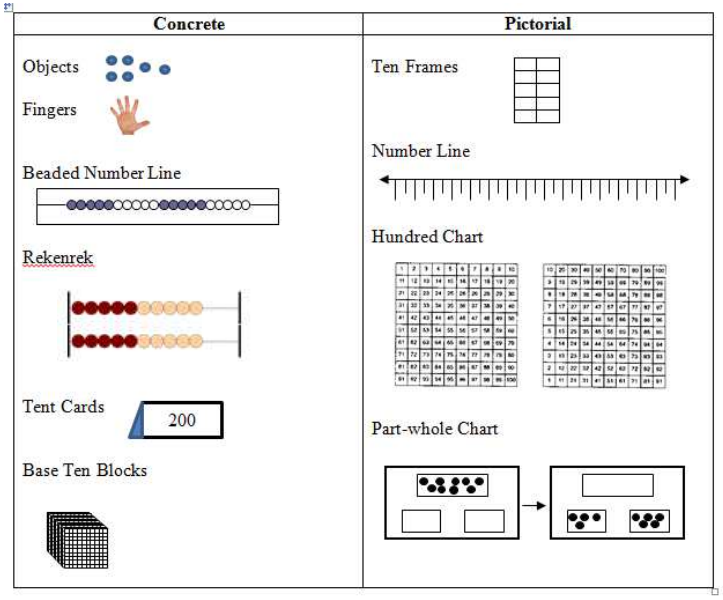
develop common assessments to assess student progress related to the benchmarks

Things to consider when teaching…Addition and Subtraction

Addition and subtraction are inverse operations. Teach them together!

- Addition is commutative (the numbers can be added in any order). For subtraction the order is important.
- When three or more numbers are added, the sum is the same, regardless of the order in which the numbers were added (associative property).
- Numbers can be partitioned in order to add or subtract.
- Place value concepts facilitate composing or decomposing numbers.
- In subtraction you can add or subtract the same amount to or from both numbers without changing the difference. (constant difference?)
- When adding or subtracting zero from a number the result is the number that you started with. (identity property)
- Adding 1 to a number or subtracting 1 from a number results in the next number (forwards or backwards) in the counting sequence.

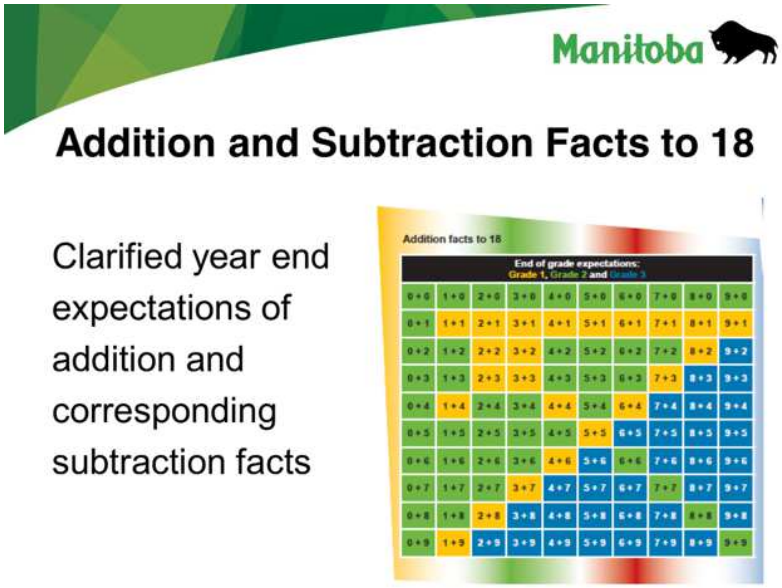
Models for Addition and Subtraction



Knowledge Domain: Addition and Subtraction

Grades 1-8

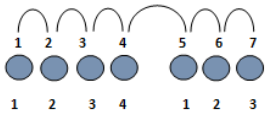
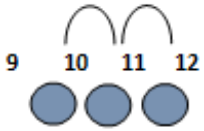
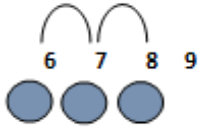
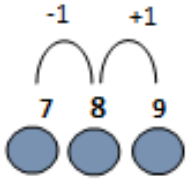
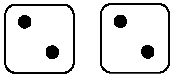
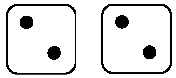
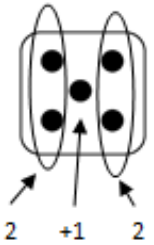
Provincial *End of Grade Level* expectations for
Addition and related Subtraction Facts

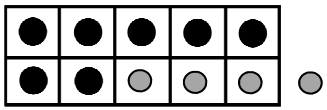
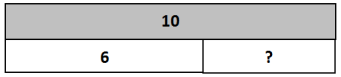
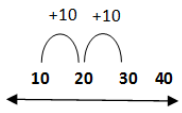
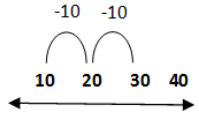


Teaching of addition and subtraction is more than just the memorization of basic facts. Basic facts are the tools for manipulating numbers easily in problem solving contexts.

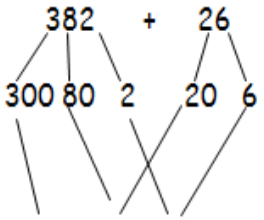
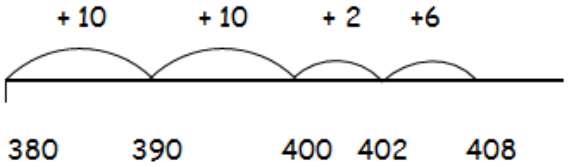
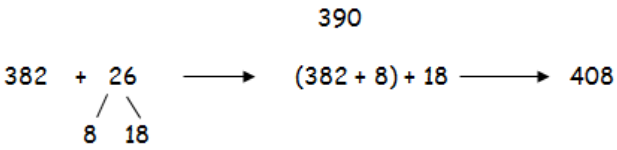
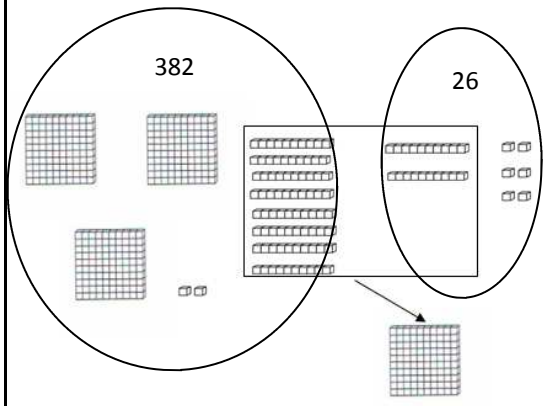
For students to understand addition and subtraction they must have acquired conceptual knowledge of the part-part-whole structure of working with numbers/quantities. **They use the "act" of combining, partitioning, and removing parts to develop this concepts of part-part-whole which lays the foundation for future mathematics instruction. This includes the knowledge of numbers and their parts, as well as numbers "nested" within other numbers.**

Early Strategies (Grades 1 specifically for operations and then Grade 2 on as mental math strategies)

	Count All	Count On, Count Back	One more , One less	Doubles	Doubles ± 1 or 2
Development/Understanding	**4+3=** Count out 4, then count out 3, then count all. (sometimes referred to as triple counting) 	**9+3=** Start at 9 and count on to 12  **9-3=** Start at 9 and count back to **6** 	One more than 8 can be written as 8+1=9 One less than 8 can be written as 8-1=7 Students should also be able to connect this to their knowledge of the rote counting sequence. 	**1+1, 2+2, 3+3…** 	Think of known double, ± 1  Can students "see" the double plus 1 in the dice pattern for 5?  Take to more advanced doubles ± 1. E.g. 6+6+1=13 Or 7+7-1=13

	Make 10	Think Addition	± 10
	7+4= 7 + 4= → 7+3=10 3 1 → 10+1=11  Students use their understanding of partitioning of numbers to make a ten and then add on leftovers. (sometimes referred to as bridging to 10)	**10-6 =** Can be thought of as 6 + ? =10  Encourage students to use their known facts. 6+ 4=10 , so 10-6=4. (inverse operation)	Connect to skip counting forwards and backwards by ten. Recording the skip counting using addition and subtraction notation will help students make this connection. E.g. 10, 20, 30 or 30, 20, 10 (10+ 10), (20 +10) (30-10), (20-10)  

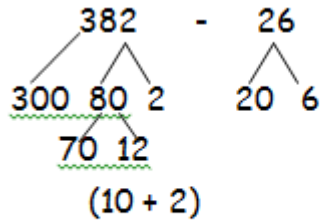
Progression in Written Methods for Addition 382 + 26

Split Strategy (Partitioning)	Empty Number Line	Compensating
This method requires place value understanding.  	(Jump Strategy)  	(Making "nice" or "friendly" numbers, bridging) 
Use representations of materials such as Base Ten Blocks	**Modified Standard Algorithm**	**Standard Algorithm (By End of Grade 4)**
 	382 + 26 8 100 300 408	382 + 26 408

Progression in Written Methods for Subtraction 382 - 26

Split Strategy (Partitioning)

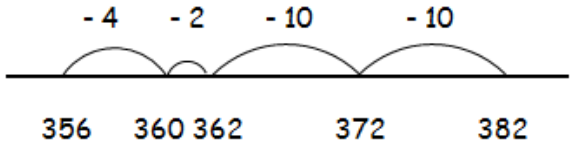
Using understanding of place value as a thinking Strategy/tool.



300 + (70 – 20) + (12 – 6) = 356

Empty Number Line

Note: There are many other strategic representations for this number sentence using the empty number line.



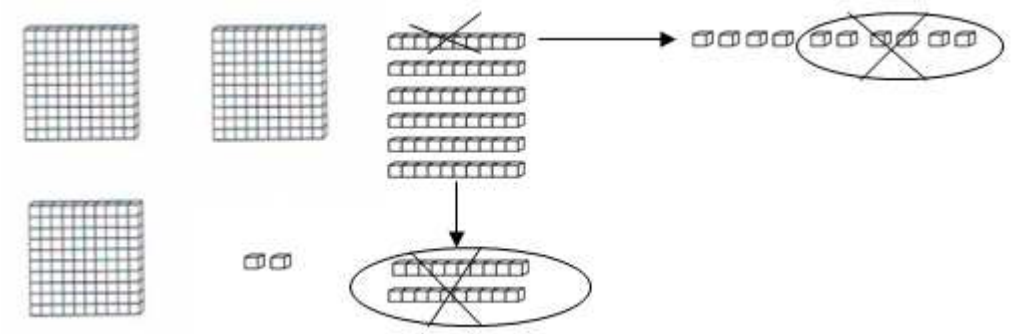
Compensating

(Making "nice" or "friendly" numbers)

Adding 4 to each number, sometimes this strategy is referred to as "constant difference" , because the difference (or distance between) both numbers remains constant.

(382 + 4) – (26 + 4) ⟶ 386 – 30 = 356

Use representations of materials such as Base Ten Blocks.



Standard Algorithm (Grade 4)

$$\begin{array}{r} 3\,{}^{7}\!\not{8}\,{}^{1}2 \\ -\quad 2\,6 \\ \hline 3\,5\,6 \end{array}$$

Students will use their own strategies.

The example below is using negative numbers.
These strategies can be used beginning in Grade 2. The size of the numbers used is determined, in part, by the outcomes related to place value.

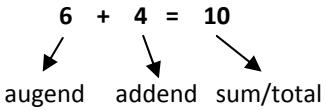
$$\begin{array}{r} 382 \\ -\quad 26 \\ \hline -\ 4\ (2 - 6) \\ 60\ (80 - 20) \\ \hline 300\ (300 - 0) \\ 356\ (300 + 60 - 4) \end{array}$$

Curricular Outcomes By Grade				
Kindergarten	**Grade 1**	**Grade 2**	**Grade 3**	**Grade 4**
Precursor to development of addition and subtraction strategies students should be exposed to activities that develop the counting principles. See counting document.	**1.N.8.**Identify the number, up to 20, that is one more, two more, one less, and two less than a given number. **1.N.9**.Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially, and symbolically by using familiar and mathematical language to describe additive and subtractive actions from their experience creating and solving problems in context that involve addition and subtraction modelling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically. **1.N.10.**Describe and use mental mathematics strategies including counting on or counting back using one more or one less making 10 starting from known doubles using addition to subtract to determine the basic addition and related subtraction facts to 18.	**2.N.8**. Demonstrate and explain the effect of adding zero to or subtracting zero from any number. **2.N.9**.Demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by using personal strategies for adding and subtracting with and without the support of manipulatives creating and solving problems that involve addition and subtraction explaining that the order in which numbers are added does not affect the sum explaining that the order in which numbers are subtracted may affect the difference. **2.N.10**.Apply mental mathematics strategies, including using doubles making 10 using one more, one less using two more, two less building on a known double using addition for subtraction to develop recall of basic addition facts to 18 and related subtraction facts.	**3.N.6**.Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as adding from left to right taking one addend to the nearest multiple of ten and then compensating using doubles. **3.N.7**.Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as taking the subtrahend to the nearest multiple of ten and then compensating thinking of addition using doubles. **3.N.8**.Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context. **3.N.9**.Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2-, and 3-digit numerals) by using personal strategies for adding and subtracting with and without the support of manipulatives creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially, and symbolically. **3.N.10.** Apply mental math strategies to determine addition facts and related subtraction facts to 18 (9 + 9).	**4.N.3** .Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) concretely, pictorially, and symbolically, by: *using personal strategies *using the standard algorithm *estimating sums and differences *solving problems **4.N.11**. Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: *using compatible numbers *estimating sums and differences *using mental math strategies to solve problems.
MB Grade Level Competency Expectations				
	Recall of one more and one less, complementary (compatible) numbers that add up to 5 and 20, doubles (up to 5 + 5) and related subtraction facts is expected by the end of Grade 1 .	Recall of facts to 10, doubles to 9 + 9, and related subtraction facts is expected by the end of Grade 2.	Recall of addition and related subtraction facts to 18 is expected by the end of Grade 3. **See MB reference chart at beginning of this document	

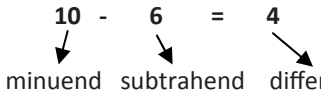
Curricular Outcomes By Grade			
Grade 5	**Grade 6**	**Grade 7**	**Grade 8**
5.N.11. Demonstrate an understanding of addition and subtraction of decimals (to thousandths), concretely, pictorially, and symbolically, by: *using personal strategies *using the standard algorithm *using estimation *solving problems	*There are no specific outcomes related to addition and subtraction at this grade level.* *Students should be solidifying their understanding of addition and subtraction in problem solving situations.* *Give students problems with different numeral ranges, including decimals.* *Students can apply their conceptual understanding and mental math strategies in the other strands.*	**7.N.2.** Demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology could be used). **7.N.5.** Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences). **7.N.6.** Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically. **7.PR.6.** Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers.	*There are no specific outcomes related to addition and subtraction at this grade level.* *Students should be solidifying their understanding of addition and subtraction in problem solving situations.* *Include problem solving situations using fractions, decimals and percent.*

Glossary of Terms

Addition - A mathematical operation of combining two or more numbers into a sum.



Subtraction - A mathematical operation that finds the difference between two quantities, or how much more one quantity is than a second quantity.



Associative Property - A property of real numbers that states that the sum or product of a set of numbers is the same, regardless of how the numbers are grouped.
Example: ***Addition***
2 + (3.5 + 1.3) = (2 + 3.5) + 1.3

Commutative Property - A property that states that an operation (addition or multiplication) is unaffected by the order in which the terms are added or multiplied.
Example: ***Addition***
The sum remains the same **2 + 3.5 = 3.5 + 2 .**

Zero Property of Addition - The property that states that the sum of a number and 0 is that same number (i.e., $a + 0 = a$ for all a). This property is sometimes called the **identity property of addition**.

Addition Types/Structures

1. **Combining** – Two or more discrete quantities are combined to form a larger quantity.
   e.g., Sam has 4 books and Mary has 3 books. How many books do they have altogether?
2. **Augmentation** – A quantity is given and something is added to it.
   e.g., Sam has 4 books and he buys 3 more. How many books does he have now?

Note: The standard algorithm is an efficient method for addition and subtraction. The algorithm is procedural and should be introduced after students have demonstrated a conceptual understanding of the operations through the use of concrete materials, visual representations and personal strategies. **In order to give students the opportunity to develop this conceptual understanding the standard algorithm is not formally introduced until Grade 4.**

Subtraction Types/Structures

Take away – A specific amount is given, a quantity is removed and what remains has to be identified.
e.g., Tom has 6 candies. He gives 3 to his sister. How many does he have left?

Part-whole (how many more) – A part of a set/ quantity is given and the amount required to reach the total has to be identified.
e.g, Pat has 4 gifts but she needs 10 in all. How many gifts should she buy?

Comparison or difference – Two quantities are matched and the difference between the two is identified.
e,g. Pete is 114 cm tall and Amy is 98 cm tall. How much taller is Pete than Amy?

Common addition and subtraction problem solving types

	Result Unknown	Change Unknown	Start Unknown	Combine	Compare (Both + and -)
Addition	**Result Unknown (a + b = ?)** Pat has 8 marbles. Her brother gives her 4. How many does she have now? (8 + 4 = ?)	**Change Unknown (a + ? = c)** Pat has 8 marbles but she would like to have 12. How many more does she need to get? (8 + ? = 12)	**Start Unknown (? + b = c)** Pat has some marbles. Her brother gave him 4 and now she has 12. How many did she have to start with? (? + 4 = 12)	**Combine (a + b = ?)** Pat has 8 blue marbles and 4 green marbles. How many does she have in all? (8 + 4 = ?)	Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have? (8 – 4 = ? or 4 + ? = 8)
Subtraction	**Result Unknown (a - b = ?)** Pat has 12 marbles. She gives her brother 4 of them. How many does she have left? (12 – 4 = ?)	**Change Unknown (a - ? = c)** Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? (12 - ? = 8)	**Start Unknown (? - b = c)** Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start with? (? – 4 = 8)	**Combine** Pat has 12 marbles. 8 are blue and the rest are green. How many are green? (12 – 8 = ?)	Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have? (8 – 4 = ? or 4 + ? = 8)

Resources

MB Documents:

- Kindergarten to Grade 8 Mathematics: Manitoba Framework of Outcomes
- Grade level support documents

Other Resources:

Big Ideas by Marian Small

Teaching Student Centered Mathematics by John Van De Walle

Web resources:

New Zealand Website , www.nzmaths .co.nz (numeracy project materials, books, Teaching Addition and Subtraction)

Further Learning:

Baroody 's Three Phase Process of Learning (2006)

Phase 1: Counting Strategies

Using objects (manipulatives) or verbal counting to determine answer

Phase 2: Reasoning Strategies

Using known information to logically determine an unknown combination

Phase 3: Producing answers efficiently (fast and accurately) "just knowing"

Retrieved August 3 2015: https;//carrieedmonds.wprdpress.com/2013/10/07/chapter-10-helping-students-master-the-basic-facts/